The Final Verdict on a Century-long Debate -- Violation of Bell's Inequality

Author: Letao Shi

October 2022

On October 4, the 2022 Nobel Prize in Physics was announced in Stockholm, Sweden. French scientist Alain Aspe, American scientist John Clauser and Austrian scientist Anton Zeilinger jointly won the Nobel Prize in Physics for their contributions to "entangled photon experiments, verification of violations of Bell's inequality and pioneering quantum information science".

Today, quantum information science is developing rapidly and has a broad and profound impact in various fields. Its origins can be traced back to quantum mechanics. Quantum mechanics was formed in the debate between Niels Bohr and Albert Einstein. Bell transformed their debate into science and proposed Bell's inequality, which provided the possibility for laboratory verification. The three scientists who won the award this time have made outstanding contributions to verifying the violation of Bell's inequality.

Next, I will talk about my understanding of Bell's inequality.

As we all know, there was a long-lasting century-long debate between Einstein and Bohr, which has a very important position in the history of physics.

This debate is based on the fundamental difference between classical mechanics and quantum mechanics. Its essence is a collision between quantum mechanics and classical mechanics. Classical mechanics is a classical theory based on Newton's laws of motion, which describes macroscopic motion that conforms to life experience. Classical mechanics believes that all objects exist objectively, and their properties and positions are predictable. The observer only "reads" their properties, and their properties will not change due to observation. Quantum mechanics believes that before an object is observed, everything is uncertain. It is the observation that causes the collapse of the wave function, that is, the observation determines the state. Due to this essential difference, the two theories are destined to be in opposition.

The most intuitive manifestation of the quantum effect is the double-slit interference experiment. In the double-slit interference experiment, photons pass through two slits and interference (interference of waves) occurs, which is manifested from a macroscopic perspective as spaced stripes appearing on the photosensitive plate. However, if only one photon is allowed to pass through at a time, interference still occurs. The explanation of quantum mechanics is that the photon has the probability of passing through the first slit and the probability of passing through the second slit, and is in a superposition state. That is to say, the photon passes through the two slits at the same time and interferes with itself. (Roger Penrose, Nobel Prize winner in Physics 2020) Observation leads to collapse, making it certain.

In 1927, at the fifth Solvay Conference, Einstein pointed out that the collapse of the wave function is inconsistent with relativity and believed that quantum mechanics is not a complete theory. Einstein believed that there was some unknown factor that affected the movement of objects and caused the results to deviate from the predictions of classical mechanics, namely the "local hidden variable theory".

At the same conference, de Broglie proposed the "guide wave theory" to explain the quantum effect. Later, after Bohm's improvement, the de Broglie-Bohm theory was finally formed. Starting from classical mechanics, the theory gives a new explanation to the quantum effect, namely: the position and momentum of the particle are determined, but a field will be emitted around it, making it know everything about the surroundings. When an electron is shot at a double slit, its quantum potential will know the existence of the two slits in advance, so that the behavior of the electron conforms to the standard interference mode. When trying to observe its position, the instrument will contact the quantum potential before the quantum potential, thus changing the behavior of the electron. However, the theory abandons locality, and quantum potential can transmit information over a distance.

In 1935, Einstein, Podolsky and Rosen proposed the EPR paradox in the article "Is quantum mechanics a complete description of physical reality?". The theory was simplified by David Bohm as follows: A system composed of two particles A and B with spin 1/2, after a certain moment, A and B separate and no longer interact. When the observer measures a certain component of A's spin, according to the conservation of angular momentum, the spin value of B in the corresponding direction can be predicted with certainty. Due to the arbitrariness of the measurement direction, the components of B's spin in all directions should be able to be predicted with certainty. The components of B's spin in all directions have certain values at the same time and exist before measurement, but quantum mechanics does not allow the simultaneous and certain prediction of the spin of particles. If we insist on considering quantum mechanics as complete, we must believe that the measurement of A can affect the state of B, thereby recognizing the existence of action at a distance.

Combined with the above text, let me talk about my understanding below.

It is known that light is an electromagnetic wave, polarized in all directions. Polaroids can filter out light in a certain direction and only allow light with the same polarization direction to pass through. When a beam of light can completely pass through polaroid a, now, place polaroid b after a, the angle is α, and its transmittance is cosα^2. If the angle α between the two polaroids is 90 degrees, its transmittance is 0. If the angle α is 22.5 degrees, its transmittance is approximately 85%. If the angle α is 45 degrees, its transmittance is approximately 50%.

However, if we put polaroid c between two mutually perpendicular polaroids a and b, and the angle between it and polaroid a is 22.5 degrees, then its transmittance is 72%. In other words, if polaroid c is placed between two mutually perpendicular polaroids a and b, as long as there is a certain angle, its transmittance will be higher and it will look brighter.

This is actually because light is an electromagnetic wave, and its polarization is a vector that can be decomposed and conforms to the parallelogram rule. Therefore, when light enters the polarizer (its polarization direction is not perpendicular to the polarizer, but there is a certain angle), it will be decomposed to form two "split beams", one of which has the same polarization direction as the polarizer and can pass through.

So in the above experiment, the light is actually decomposed when passing through polarizer a, and enters polarizer c with another polarization direction. After being decomposed again, part of it passes through polarizer b. Therefore, the two processes of light passing through polarizers a, c and passing through polarizers c, b are actually relatively independent.

Then, according to this phenomenon when light passes through the polarizer, physicists designed an ingenious experiment. First, prepare a pair of entangled particles with 0 angular momentum. At a certain moment, they separate and fly in two opposite directions. According to the law of conservation of angular momentum in classical mechanics, if we measure the state of one particle a, we can know the state of the other particle b. But in quantum mechanics, the states of these two particles are in a superposition state before measurement, and their states cannot be determined. Due to measurement, they collapse into the state we see.

Now, suppose the two particles are far enough apart that they cannot interfere with each other. If the state of particle a is measured. Then, according to the law of conservation of angular momentum, the state of particle b is also determined. Now, we measure the states of particles a and b at both ends.

Assume that the angle between the two polarizers is not 0 degrees or 90 degrees, but α. (The following takes α=22.5 degrees as an example) When the probability of particle a passing through a is 85%, according to the principles of classical mechanics, the probability of particle b passing through is 50%. However, according to the principles of quantum mechanics, the state of particle a is determined when it is measured. After it is "decomposed" by the polarizer to change its polarization direction, the polarization direction of particle b also changes in an instant, causing the probability of it passing through the polarizer to change. In other words, particles a and b know each other's probability of passing through the polarizer. Therefore, by calculation, the probability of particle b passing through the polarizer is 72%, which is much greater than 50%.

Based on this phenomenon, Bell derived an inequality, the Bell inequality, by simply calculating the probabilities of passing through different particle states. If the local hidden variable theory is valid, then after a certain calculation of the probability of the particle passing in each direction, the result is less than or equal to 2. However, according to the interpretation of quantum mechanics, the maximum result can reach $2\sqrt{2}$, which is approximately equal to 2.848.

Then, verifying whether Bell's inequality is valid has become a touchstone for whether the hidden variable theory and quantum mechanics are complete.

In 1972, John Clauss excited a pair of entangled photons through calcium atoms, and after recording the probabilities of them passing through the polarizers respectively, he found that the results did not conform to Bell's inequality. In other words, Bell's inequality does not hold. However. There are still many problems in this experiment, such as the experimental results are not accurately measured, the direction of the polarizer is fixed, etc. The most important thing is that the detection points on both sides are very close, and the photons may "communicate" with each other without violating locality, and there is a "communication loophole". Although the reality is falsified, the non-locality of quantum entanglement cannot be proved. Therefore, the results of this test are not enough to convince everyone.

So, in 1982, Alan Aspe improved the above experiment. He avoided the possibility of "communication" between the two photons by quickly and irregularly changing the trajectory of the photons so that they passed through polarizers in different directions. However, the single-photon detector used in the experiment had measurement errors, so it was still not absolutely convincing.

Finally, in 2015, Anton Zeilinger designed and implemented a perfect experiment. He measured the entangled diamond color center electrons in two laboratories 1.3 kilometers apart, and successfully falsified the locality of quantum entanglement while ensuring experimental accuracy, completely overturning Bell's inequality and declaring the error of the local hidden variable theory with facts.

Since then, the century-long debate on quantum mechanics and local hidden variable theory has finally come to an end. Facts have proved that quantum mechanics is a complete theory, thus laying the foundation for future developments in quantum information theory.

Appendix:

It is known that when measuring the spin of an electron, the result can be either up or down. Measuring along three directions yields three data points. Suppose electrons A and B are entangled. If A's spin in the X-direction is up, then B's must be down.

We define: if the spins of A and B are the same, their correlation is +1. If opposite, the correlation is -1.

According to classical mechanics, the probabilities can be represented as:

$P_{xy} = -N1 - N2 + N3 + N4 + N5 + N6 - N7 - N8;$
$P_{zy} = -N1 + N2 + N3 - N4 - N5 + N6 + N7 - N8;$
$P_{xz} = -N1 + N2 - N3 + N4 + N5 - N6 + N7 - N8;$

Then:

$|P_{xz} - P_{zy}| = |-2N3 + 2N4 + 2N5 - 2N6| = 2|(N4 + N5) - (N3 + N6)| \leq 2[|N4 + N5| + |N3 + N6|];$

Given the total probability sum is 1, i.e.,

$N1 + N2 + N3 + N4 + N5 + N6 + N7 + N8 = 1;$

Then:

$|P_{xz} - P_{zy}| \leq (N3 + N4 + N5 + N6) + (1 - N1 - N2 - N7 - N8);$

Thus:

$|P_{xz} - P_{zy}| \leq 1 + P_{xy};$

This essentially addresses the question: Does measuring electron A affect the state of electron B, and can such influence disregard the constraints of space and time?

Ax	Ay	Az	Bx	By	Bz	Possibility
+	+	+	-	-	-	N1
+	+	-	-	-	+	N2
+	-	+	-	+	-	N3
+	-	-	-	+	+	N4
-	+	+	+	-	-	N5
-	+	-	+	-	+	N6
-	-	+	+	+	-	N7
-	-	-	+	+	+	N8